

Anderson-Darling Type Goodness-of-fit Statistic Based on a Multifold Integrated Empirical Distribution Function

Satoshi Kuriki

Institute of Statistical Mathematics, Japan

Hsien-Kuei Hwang

Institute of Statistical Science, Academia Sinica, Taiwan

Abstract

An Anderson-Darling type goodness-of-fit statistic constructed from multifold integrated empirical distribution function is proposed. Our proposed statistic is of an integral form whose integrand is a standardized square of m -fold integrated empirical distribution function. The empirical distribution functions is adjusted in advance so that it does not contain the components of m -th or lower degree polynomials. Our proposed statistic is a natural extension of the goodness-of-fit statistic by Anderson and Darling (1952, AMS), which corresponds to the case $m=1$. When $m=2$, our proposed statistic has much statistical power to detect the discrepancy of dispersion of distribution. The Karhunen-Loeve expansion of the limiting integrand process is obtained with Legendre eigenfunctions, and the limiting distribution of our propose statistic is proved to be a weighted sum of chi-square random variables with the weights $1/\{k(k+1)\dots(k+2m-1)\}$, $k=1,2,\dots$. The explicit form of the Laplace transform of the limiting distribution is also derived.